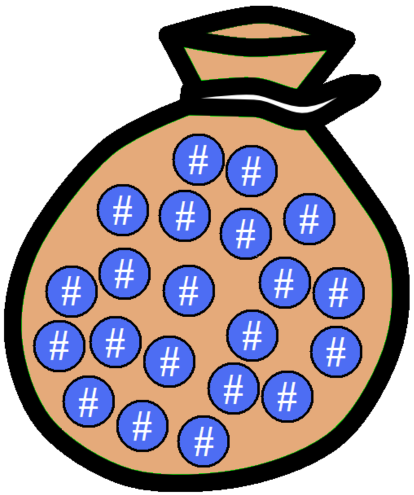
Sec. 9.3: Building Confidence Intervals for a Population Standard Deviation σ or a Population Variance σ^2

Idea Behind Section 9.3

Population Data (unknown)



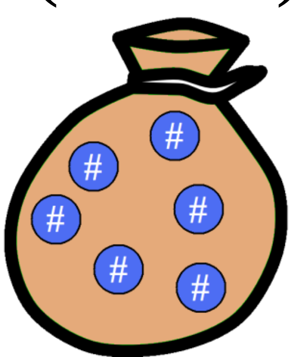
Population Parameter (unknown)

$$\sigma = ?$$

or

$$\sigma^2 = ?$$

Sample Data (known)



Sample Statistic (known)

$$s$$

or

$$s^2$$

- Goal is to estimate the population standard deviation σ (or population variance σ^2)
- To estimate this, take a sample and calculate the sample standard deviation s (or sample variance s^2)
- s (or s^2) is the best point estimate of σ (or σ^2) respectively
- Now build an interval around s (or s^2) to get your confidence interval

Why Do We Care About The Population Standard Deviation σ (or Population Variance σ^2)?

- Population standard deviations and variances tell us about consistency

Note:

Make sure to read the problems carefully! (Why?)

Point Estimates vs. Interval Estimates

Point Estimate

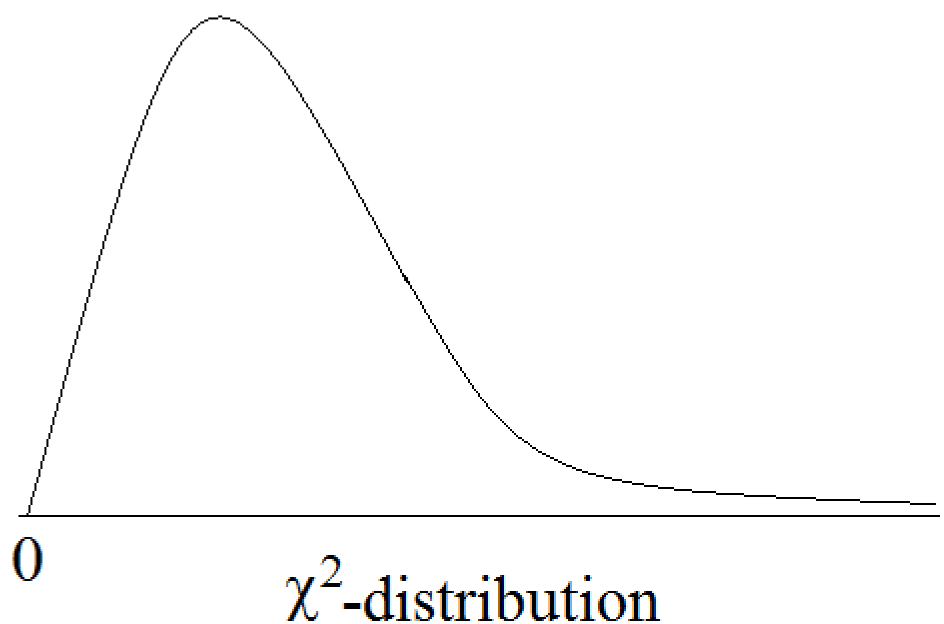
- Best point estimate of σ is s (and the best point estimate of σ^2 is s^2)
- Point estimates are bad because they are only 1 number estimates and almost always wrong

Interval Estimate

- We want an interval estimate…a range of values so that our answer has a better chance of being correct
- Don’t want the interval to be too big (wide)
- Every interval estimate has a probability attached to it (called its confidence level).
- The interval is called a confidence interval
- The confidence level tells you the probability that the correct answer for σ (or σ^2) is in your interval.

Things To Know About The χ^2 - Distributions

- χ^2 (Chi-Squared) is the entire name of the random variable. Nothing is being squared here, the square is just part of the name. This is not the square of another random variable χ. There is no random variable χ
- There are many χ^2-distributions, one for each sample size (or degree of freedom)
- χ^2-distributions are not symmetric. They start at 0 and have a tail going to the right forever
- All χ^2 -distributions look like



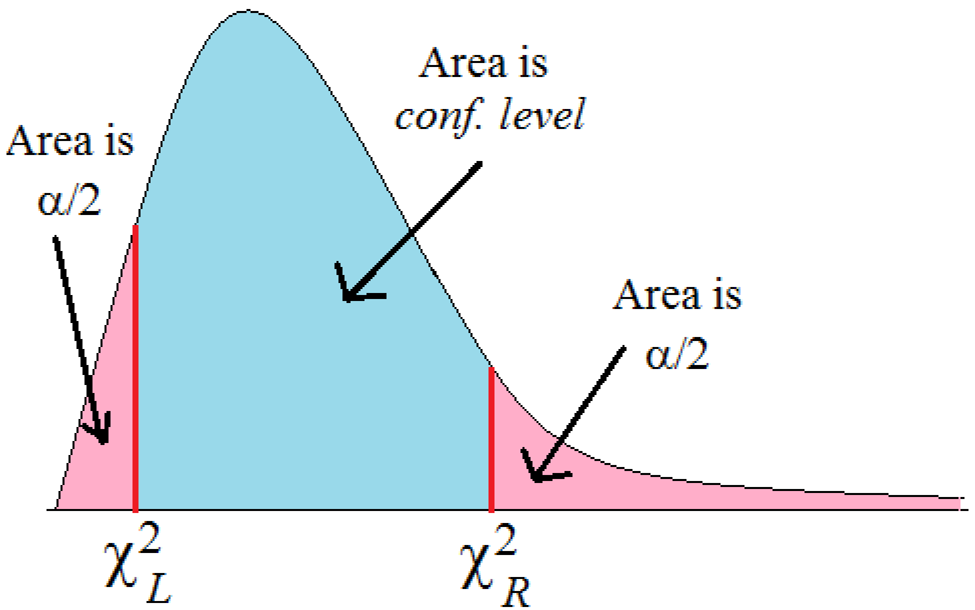
- Possible values of χ^2 are $[0, \infty)$

Things To Know About The χ^2 - Distributions

- When building confidence intervals for σ (or σ^2), you'll need to look up 2 numbers from the χ^2 table: $\chi^2{}_L$ and $\chi^2{}_R$

  To find these numbers:

  1. Find the degrees of freedom $df = n - 1$
  2. Draw a picture like …



- Degrees of freedom are on left of table (if not there, use the closest one), areas to right are on top (if not there, use the closest one), and values of $\chi^2{}_L$ and $\chi^2{}_R$ are in the middle of the table

Building Confidence Intervals for a Population Standard Deviation σ (or a Population Variance σ^2)

Quantity you are trying to estimate: σ (or σ^2)

Best point estimate: s (or s^2)

Formula for alpha: $\propto = 1 - conf.\ level$

Degrees of freedom: $df = n - 1$

Building Confidence Intervals for a Population Standard Deviation σ (or a Population Variance σ^2)

Confidence Interval for σ: $\sqrt{\dfrac{(n-1)s^2}{{\chi^2}_R}} < \sigma < \sqrt{\dfrac{(n-1)s^2}{{\chi^2}_L}}$

Confidence Interval for σ^2: $\dfrac{(n-1)s^2}{{\chi^2}_R} < \sigma^2 < \dfrac{(n-1)s^2}{{\chi^2}_L}$

Condition: Population from which samples are drawn have a NORMAL distribution

Building Confidence Intervals for a Population Standard Deviation σ (or a Population Variance σ^2)

Notes:

- Confidence intervals for σ (or σ^2) are NOT constructed by finding a margin or error *E* and adding and subtracting it from the sample statistic s (or s^2). Instead, just use the given formula
- Confidence Intervals for σ (or σ^2) are not symmetric about the sample statistics s (or s^2)

Ex 1 (Sec. 9.3 Hw #15 pg. 460): **Peanuts** A jar of peanuts is supposed to have 16 ounces of peanuts. The filling machine inevitably experiences fluctuations in filling, so a quality control manager randomly samples 12 jars of peanuts from the storage facility and measured their contents in order to estimate the standard deviation of the number of ounces of peanuts in all jars filled by this machine. She obtains the following data:

Ounces of Peanuts							
15.94	15.74	16.21	15.36	15.84	15.84	n =	
15.52	16.16	15.78	15.51	16.28	16.53	xbar =	
						s =	

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a) What is the population?
b) What is the sample?
c) What is the population parameter (symbol and description)?
d) What is your sample statistic (symbol, description, and value)?
e) What is your best point estimate for the population parameter?

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f) Find your 90% confidence interval for the population parameter

g) What does the 90% in a 90% confidence interval mean?

h) The quality control manager wants the machine to have a population standard deviation below 0.20 ounce. Does the confidence interval validate this desire?

Ex 2 (Sec. 9.3 Hw #16 pg. 460): **Investment Risk** Investors not only desire a high return on their money, but they would also like the rate of return to be stable from year to year. An investment manager invests with the goal of reducing volatility (year-to-year fluctuations in the rate of return). The following data represent the rate of return (in percent) for his mutual fund for the past 12 years. The investor wants to estimate the standard deviation of the return rates for his mutual fund for all years.

Mutual Fund Rate of Return							
13.8	15.9	10	12.4	11.3	6.6	n =	12
9.6	12.4	10.3	8.7	14.9	6.7	xbar =	11.05
						s =	2.97917

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f) Find your 95% confidence interval for the population parameter

g) What does the 95% in a 95% confidence interval mean?

h) The investment manager wants to have a population standard deviation for the rate of return below 6%. Does the confidence interval validate this desire?